Note on the strong law of large numbers in a Hilbert space [1]

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Abstract

We study the Kolmogorov's strong law of large numbers for the sums of Hilbert valued random variables under the condition $E\|X_1\| < \infty$ and the weaker assumption that the random variables is only pairwise independent identically distributed. In addition, the strong law of large numbers for r-dimensional arrays is also obtained.

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