

The Fekete-Szegö Theorem for Certain Class of Analytic Functions ¹

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Abstract

In this paper we discuss a well-known class studied by Ramesha [12] and later by several authors (see [8, 1, 7]). Next we extend the class to a wider class of functions f denoted by $u_{\alpha,\beta}^\gamma$, which are normalised and univalent, in the open unit disc $D = \{z \in \mathbb{C} : |z| < 1\}$ satisfying the condition

$$\operatorname{Re} \left(\frac{\alpha z^2 f''(z)}{g(z)} + \frac{z f'(z)}{g(z)} \right) > \beta, \quad 0 \leq \alpha < 1 \text{ and } 0 \leq \beta < 1,$$

where $g \in S^*(\gamma)$, $g(z) \neq 0$ and $0 \leq \gamma < 1$, is a normalised starlike function of order γ . For $f \in u_{\alpha,\beta}^\gamma$ we shall obtain sharp bounds for the Fekete-Szegö functional $|a_3 - \mu a_2^2|$ when μ is real.

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