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Subclasses of $\alpha-$uniformly convex functions obtained by using an integral operator and the theory of strong differential subordinations [1]

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Abstract

In this paper we define some subclasses of $\alpha-$uniformly convex functions with respect to a convex domain included in the right half plane D, obtained by using an integral operator and the theory of strong differential subordinations. The notion of strong differential subordination is developed from the classic notion of differential subordination.

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1 Introduction

Let U denote the unit disc of the complex plane :

$$U = \{z \in \mathbb{C} : \ |z| < 1\}$$

and

$$\overline{U} = \{z \in \mathbb{C} : \ |z| \le 1\}.$$

Let $\mathcal{H}(U \times \overline{U})$ denote the class of analytic functions in $U \times \overline{U}$.
In [10], the authors have defined the class

$$\mathcal{H}\zeta[a, n] = \{f \in \mathcal{H}(U \times \overline{U}) : \ f(z, \zeta) = a + a_n(\zeta)z^n + a_{n+1}(\zeta)z^{n+1} + \cdots, \ z \in U, \zeta \in \overline{U}\}$$

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with $a_k(\zeta)$ holomorphic functions in $\overline{U}$, $k \geq n$,

$$\mathcal{H}\zeta_u(U) = \{f \in \mathcal{H}\zeta[a,n]: \ f(z,\zeta) \text{ univalent in } \ U \times \overline{U}, \ z \in U, \ \text{for all} \, \zeta \in \overline{U}\},$$

$$\mathcal{A}\zeta_n = \{f \in \mathcal{H}\zeta[a,n]: \ f(z,\zeta) = z + a_2(\zeta)z^2 + \cdots + a_n(\zeta)z^n + \cdots, \ z \in U, \zeta \in \overline{U}\}$$

with $\mathcal{A}\zeta_1 = \mathcal{A}\zeta$,
and

$$\mathcal{S}\zeta = \{f \in \mathcal{A}\zeta: \ f(z,\zeta) \text{ univalent in } \ U \times \overline{U}, \ z \in U, \ \text{for all} \, \zeta \in \overline{U}\}.$$

Let

$$S^*\zeta = \left\{f \in \mathcal{A}\zeta: \ \mathrm{Re}\, \frac{zf'(z,\zeta)}{f(z,\zeta)} > 0, \ z \in U, \ \text{for all} \, \zeta \in \overline{U}\right\}$$

denote the class of starlike functions in $U \times \overline{U}$,

$$K\zeta = \left\{f \in \mathcal{A}\zeta: \ \mathrm{Re}\, \frac{zf''(z,\zeta)}{f'(z,\zeta)} + 1 > 0, \ z \in U, \ \text{for all} \, \zeta \in \overline{U}\right\}$$

denote the class of normalized convex functions in $U \times \overline{U}$, and

$$C\zeta = \left\{f \in \mathcal{A}\zeta: \ \exists \varphi \in K\zeta, \mathrm{Re}\, \frac{f'(z,\zeta)}{\varphi'(z,\zeta)} > 0, \ z \in U, \ \text{for all} \, \zeta \in \overline{U}\right\}$$

denote the class of close-to-convex functions in $U \times \overline{U}$.

Definition 1 *[10] Let $h(z,\zeta)$, $f(z,\zeta)$ be analytic in $U \times \overline{U}$. The function $f(z,\zeta)$ is said to be strongly subordinate to $h(z,\zeta)$, or $h(z,\zeta)$ is said to be strongly superordinate to $f(z,\zeta)$, if there exists a function ω analytic in U, $\omega(0) = 0$, $|\omega(z)| < 1$, such that $f(z,\zeta) = h[\omega(z),\zeta]$, for all $\zeta \in \overline{U}$.*
In such a case we write $f(z,\zeta) \prec\prec h(z,\zeta), z \in U, \zeta \in \overline{U}$.

Remark 1 *(i) If $h(z,\zeta) \equiv h(z)$ and $f(z,\zeta) \equiv f(z)$, then the strong subordination becomes the usual notion of subordination.*
(ii) The notion of differential subordination was introduced and developed by S. S. Miller and P. T. Mocanu [6], and the concept of strong differential subordination was introduced in [2] by J. A. Antonino and S. Romaguera and developed by G. I. Oros and Gh. Oros [10].

Definition 2 *[3] Let consider the integral operator $L_a: \mathcal{A}\zeta_n \to \mathcal{A}\zeta_n$ defined as:*

$$f(z,\zeta) = L_a F(z,\zeta) = \frac{1+a}{z^a} \int_0^z F(t,\zeta) t^{a-1} dt, \quad a \in \mathbb{C}, \quad \mathrm{Re}\, a \geq 0. \tag{1}$$

In the case $a = 1, 2, 3, \cdots$ this operator was introduced by S. D. Bernardi [3], and it was studied by many authors.

Definition 3 *[12] For $f(z,\zeta) \in \mathcal{A}\zeta_n$, $n \in \mathbb{N}^* \cup \{0\}$, we define the differential operator $D^n : \mathcal{A}\zeta_n \to \mathcal{A}\zeta_n$*

$$\begin{aligned} &D^0 f(z,\zeta) = f(z,\zeta)\\ &D^1 f(z,\zeta) = zf'(z,\zeta)\\ &\ldots\\ &D^{n+1} f(z,\zeta) = z[D^n f(z,\zeta)]', \quad z \in U, \zeta \in \overline{U}. \end{aligned}$$

We note that the derivative is to respect to the first variable. For $f(z) \in \mathcal{H}(U)$ we have Sălăgean differential operator [12].

Proposition 1 *For $f(z,\zeta) \in \mathcal{A}\zeta_n$, $n \in \mathbb{N}^* \cup \{0\}$, with the differential operator $D^n : \mathcal{A}\zeta_n \to \mathcal{A}\zeta_n$ we have:*

$$z[D^{n+1} f(z,\zeta)]' = D^n f(z,\zeta), \quad z \in U, \zeta \in \overline{U}.$$

2 Preliminary results

The next two definitions (given in [5]) are adapted to the class $\mathcal{H}\zeta[a,n]$:

Definition 4 *[5] Let $\alpha \in [0,1]$ and $f(z,\zeta) \in \mathcal{A}\zeta_n$. We say that f is a $\alpha-$uniformly convex function if*

$$\mathrm{Re}\left[(1-\alpha)\frac{zf'(z,\zeta)}{f(z,\zeta)} + \alpha(1+\frac{zf''(z,\zeta)}{f'(z,\zeta)})\right] \geq \left|(1-\alpha)(\frac{zf'(z,\zeta)}{f(z,\zeta)} - 1) + \alpha\frac{zf''(z,\zeta)}{f'(z,\zeta)}\right|,$$

$z \in U, \zeta \in \overline{U}$.
We denote this class by $UM\zeta_\alpha$.

Remark 2 *Geometric interpretation: $f \in UM\zeta_\alpha$ if and only if*

$$J(\alpha, f; z, \zeta) = (1-\alpha)\frac{zf'(z,\zeta)}{f(z,\zeta)} + \alpha(1+\frac{zf''(z,\zeta)}{f'(z,\zeta)})$$

take all values in the parabolic region $\Omega = \{w : |w-1| \leq \mathrm{Re}\, w\} = \{w = u + \imath v : v^2 \leq 2u-1\}$. For a fixed ζ we obtain $UM_0 = SP$, where the class SP was introduced by F. Ronning in [11] and $UM_\alpha \subset M_\alpha$, where M_α is the well know class of $\alpha-$ convex functions introduced by P. T. Mocanu in [9].

Definition 5 *[5] Let $\alpha \in [0,1]$ and $n \in \mathbb{N}$. We say that $f(z,\zeta) \in \mathcal{A}\zeta_n$ is in the class $UD\zeta_{n,\alpha}(\beta,\gamma)$, $\beta \geq 0$, $\gamma \in [-1,1)$, $\beta + \gamma \geq 0$ if*

$$\mathrm{Re}\left[(1-\alpha)\frac{D^{n+1} f(z,\zeta)}{D^n f(z,\zeta)} + \alpha\frac{D^{n+2} f(z,\zeta)}{D^{n+1} f(z,\zeta)}\right] \geq$$

$$\geq \beta\left|(1-\alpha)\frac{D^{n+1} f(z,\zeta)}{D^n f(z,\zeta)} + \alpha\frac{D^{n+2} f(z,\zeta)}{D^{n+1} f(z,\zeta)} - 1\right| + \gamma.$$

Remark 3 *Geometric interpretation:* $f \in UD\zeta_{n,\alpha}(\beta,\gamma)$ *if and only if*

$$J_n(\alpha, f; z, \zeta) = (1-\alpha)\frac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} + \alpha\frac{D^{n+2}f(z,\zeta)}{D^{n+1}f(z,\zeta)}$$

takes all values in the convex domain included in right half plane $D_{\beta,\gamma}$*, where* $D_{\beta,\gamma}$ *is a elliptic region for* $\beta > 1$*, a parabolic region for* $\beta = 1$*, a hyperbolic region for* $0 < \beta < 1$*, the half plane for* $\beta = 0$*. We have* $UD\zeta_{0,\alpha}(1,0) = UM\zeta_\alpha$*.*

The next theorem is a result due to so called "admissible functions method" introduced by P. T. Mocanu and S. S. Miller (see [6], [7], [8]) and adapted to the class $\mathcal{H}\zeta[a,n]$.

Theorem 1 *[6], [7], [8] Let* $h \in K\zeta$ *and* $\mathrm{Re}\,[\beta h(z,\zeta) + \delta] > 0$*,* $z \in U, \zeta \in \overline{U}$*. If* $p \in \mathcal{H}(U \times \overline{U})$ *with* $p(0,\zeta) = h(0,\zeta)$ *and* p *satisfies the Briot-Bouquet strong differential subordination*

$$p(z,\zeta) + \frac{zp'(z,\zeta)}{\beta p(z,\zeta) + \delta} \prec\prec h(z,\zeta),$$

then $p(z,\zeta) \prec\prec h(z,\zeta)$*.*

The next definition (given in [4]) is adapted to the class $\mathcal{H}\zeta[a,n]$.

Definition 6 *[4] The function* $f(z,\zeta) \in \mathcal{A}\zeta_n$ *is n-starlike with respect to convex domain included in right half plane* D *if the differential expression* $\dfrac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)}$ *takes values in the domain* D*.*

Remark 4 *If we consider* $q(z,\zeta)$ *an univalent function with* $q(0,\zeta) = 1$*,* $\mathrm{Re}\,q(z,\zeta) > 0$*,* $q'(0,\zeta) > 0$*, which maps the unit disc* U *into the convex domain* D*, we have:*

$$\frac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} \prec\prec q(z,\zeta).$$

We denote by $S^*\zeta_n(q)$ *the class of all these functions.*

3 Main results

Let $q(z,\zeta)$ be an univalent function with $q(0,\zeta) = 1$, $q'(0,\zeta) > 0$, which maps the unit disc U into a convex domain included in right half plane D.

The next definition (given in [1]) is adapted to the class $\mathcal{H}\zeta[a,n]$.

Definition 7 *[1] Let* $f(z,\zeta) \in \mathcal{A}\zeta_n$ *and* $\alpha \in [0,1]$*. We say that* f *is a* $\alpha-$*uniform convex function with respect to* D*, if*

$$J(\alpha, f; z, \zeta) = (1-\alpha)\frac{zf'(z,\zeta)}{f(z,\zeta)} + \alpha(1 + \frac{zf''(z,\zeta)}{f'(z,\zeta)}) \prec\prec q(z,\zeta).$$

We denote this class by $UM\zeta_\alpha(q)$*.*

Remark 5 *Geometric interpretation:* $f \in UM\zeta_\alpha(q)$ *if and only if* $J(\alpha, f; z, \zeta)$ *takes all values in the convex domain included in right half plane* D.

Remark 6 *If we take* $D = \Omega$ *(see Remark 2) we obtain the class* $UM\zeta_\alpha$.

Remark 7 *From the above definition it easily results that* $q_1(z,\zeta) \prec\prec q_2(z,\zeta)$ *implies* $UM\zeta_\alpha(q_1) \subset UM\zeta_\alpha(q_2)$.

Theorem 2 *For all* $\alpha, \alpha' \in [0,1]$, *with* $\alpha < \alpha'$, *we have* $UM\zeta_{\alpha'}(q) \subset UM\zeta_\alpha(q)$.

Proof. From $f \in UM\zeta_{\alpha'}(q)$ we have

$$(2) \qquad J(\alpha', f; z, \zeta) = (1-\alpha')\frac{zf'(z,\zeta)}{f(z,\zeta)} + \alpha'(1 + \frac{zf''(z,\zeta)}{f'(z,\zeta)}) \prec\prec q(z,\zeta),$$

where $q(z,\zeta)$ is univalent in U with $q(0,\zeta) = 1$, $q'(0,\zeta) > 0$, and maps the unit disc U into the convex domain included in right half plane D.
With the notation $\dfrac{zf'(z,\zeta)}{f(z,\zeta)} = p(z,\zeta)$, where

$$p(z,\zeta) = 1 + p_1(z,\zeta) + \cdots, \quad z \in U, \zeta \in \overline{U},$$

we obtain:

$$J(\alpha', f; z, \zeta) = p(z,\zeta) + \alpha' \frac{zp'(z,\zeta)}{p(z,\zeta)}.$$

From (2) we have

$$p(z,\zeta) + \alpha' \frac{zp'(z,\zeta)}{p(z,\zeta)} \prec\prec q(z,\zeta),$$

with $p(0,\zeta) = q(0,\zeta)$, $\operatorname{Re} q(z,\zeta) > 0$, $z \in U, \zeta \in \overline{U}$.
In these conditions from Theorem 1, with $\delta = 0$, we obtain $p(z,\zeta) \prec\prec q(z,\zeta)$, or $p(z,\zeta)$ takes all values in D.
If we consider the function $g : [0,\alpha'] \times \overline{U} \to \mathbb{C}$,

$$g(u,\zeta) = p(z,\zeta) + u\frac{zp'(z,\zeta)}{p(z,\zeta)},$$

with $g(0,\zeta) = p(z,\zeta) \in D$ and $g(\alpha',\zeta) = J(\alpha', f; z, \zeta) \in D$, since the geometric image of $g(\alpha,\zeta)$ is on the segment obtained by the union of the geometric image of $g(0,\zeta)$ and $g(\alpha',\zeta)$, we have $g(\alpha,\zeta) \in D$ or $p(z,\zeta) + \alpha\dfrac{zp'(z,\zeta)}{p(z,\zeta)} \in D$.
Thus $J(\alpha, f; z, \zeta)$ takes all values in D, or $J(\alpha, f; z, \zeta) \prec\prec q(z,\zeta)$. This means $f \in UM\zeta_\alpha(q)$.

Theorem 3 *If* $F(z,\zeta) \in UM\zeta_\alpha(q)$ *then* $f(z,\zeta) = L_aF(z,\zeta) \in S^*\zeta_0(q)$, *where* L_a *is the integral operator defined by* (1) *and* $\alpha \in [0,1]$.

Proof. From (1) we have

$$(1+a)F(z,\zeta) = af(z,\zeta) + zf'(z,\zeta), \quad z \in U, \zeta \in \overline{U}.$$

With the notation $\dfrac{zf'(z,\zeta)}{f(z,\zeta)} = p(z,\zeta)$, where

$$p(z,\zeta) = 1 + p_1(z,\zeta) + \cdots, \quad z \in U, \zeta \in \overline{U},$$

we have:

$$\frac{zF'(z,\zeta)}{F(z,\zeta)} = p(z,\zeta) + \frac{zp'(z,\zeta)}{p(z,\zeta)+a}.$$

If we denote $\dfrac{zF'(z,\zeta)}{F(z,\zeta)} = h(z,\zeta)$, with $h(0,\zeta) = 1$, we have from $F(z,\zeta) \in UM\zeta_\alpha(q)$ (see Definition 7) that:

$$h(z,\zeta) + \alpha\frac{zh'(z,\zeta)}{h(z,\zeta)} \prec\prec q(z,\zeta),$$

where $q(z,\zeta)$ is univalent in U with $q(0,\zeta) = 1$, $q'(z,\zeta) > 0$, and maps the unit disc U into the convex domain included in right half plane D.
From Theorem 1 we obtain
$h(z,\zeta) \prec\prec q(z,\zeta)$ or $p(z,\zeta) + \dfrac{zp'(z,\zeta)}{p(z,\zeta)+a} \prec\prec q(z,\zeta)$.
Using the hypothesis and the construction of the function $q(z,\zeta)$ we obtain from Theorem 1 that
$\dfrac{zf'(z,\zeta)}{f(z,\zeta)} = p(z,\zeta) \prec\prec q(z,\zeta)$ or $f(z,\zeta) \in S^*\zeta_0(q) \subset \mathcal{S}^*\zeta$.

The next definition (given in [1]) is adapted to the class $\mathcal{H}\zeta[a,n]$.

Definition 8 *[1] Let $f(z,\zeta) \in \mathcal{A}\zeta_n$ and $\alpha \in [0,1]$, $n \in \mathbb{N}$. We say that f is an $\alpha - n-$uniformly convex function with respect to D, if*

$$J_n(\alpha, f; z,\zeta) = (1-\alpha)\frac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} + \alpha\frac{D^{n+2}f(z,\zeta)}{D^{n+1}f(z,\zeta)} \prec\prec q(z,\zeta).$$

We denote this class by $UD\zeta_{n,\alpha}(q)$.

Remark 8 *Geometric interpretation: $f \in UD\zeta_{n,\alpha}(q)$ if and only if $J_n(\alpha, f; z,\zeta)$ takes all values in the convex domain included in right half plane D.*

Remark 9 *If we consider $D = D_{\beta,\gamma}$ (see Remark 3) we obtain the class $UD\zeta_{n,\alpha}(\beta,\gamma)$.*

Remark 10 *From the above definition it easily results that $q_1(z,\zeta) \prec\prec q_2(z,\zeta)$ implies $UD\zeta_{n,\alpha}(q_1) \subset UD\zeta_{n,\alpha}(q_2)$.*

Theorem 4 *For all $\alpha, \alpha' \in [0,1]$, with $\alpha < \alpha'$, we have*

$$UD\zeta_{n,\alpha'}(q) \subset UD\zeta_{n,\alpha}(q).$$

Proof. From $f \in UD\zeta_{n,\alpha'}(q)$ we have

$$J_n(\alpha', f; z, \zeta) = (1-\alpha')\frac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} + \alpha' \frac{D^{n+2}f(z,\zeta)}{D^{n+1}f(z,\zeta)} \prec\prec q(z,\zeta), \tag{3}$$

where $q(z,\zeta)$ is univalent in U with $q(0,\zeta)=1$, $q'(0,\zeta)>0$, and maps the unit disc U into the convex domain included in right half plane D.
With the notation $\dfrac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} = p(z,\zeta)$, where

$$p(z,\zeta) = 1 + p_1(z,\zeta) + \cdots, \quad z \in U, \zeta \in \overline{U},$$

we have:

$$J_n(\alpha', f; z, \zeta) = p(z,\zeta) + \alpha' \frac{zp'(z,\zeta)}{p(z,\zeta)}.$$

From (3) we obtain

$$p(z,\zeta) + \alpha' \frac{zp'(z,\zeta)}{p(z,\zeta)} \prec\prec q(z,\zeta)$$

with $p(0,\zeta) = q(0,\zeta)$, $\operatorname{Re} q(z,\zeta) > 0$, $z \in U, \zeta \in \overline{U}$.
In these conditions from Theorem 1 we obtain $p(z,\zeta) \prec\prec q(z,\zeta)$, or $p(z,\zeta)$ takes all values in D.
If we consider the function $g : [0,\alpha'] \times \overline{U} \to \mathbb{C} \times \overline{U}$,

$$g(u,\zeta) = p(z,\zeta) + u\frac{zp'(z,\zeta)}{p(z,\zeta)},$$

with $g(0,\zeta) = p(z,\zeta) \in D$ and $g(\alpha',\zeta) = J_n(\alpha', f; z, \zeta) \in D$, it is easy to see that

$$g(\alpha,\zeta) = p(z,\zeta) + \alpha\frac{zp'(z,\zeta)}{p(z,\zeta)} \in D.$$

Thus we have $J_n(\alpha, f; z, \zeta) \prec\prec q(z,\zeta)$ or $f \in UD\zeta_{n,\alpha}(q)$.

Theorem 5 *If $F(z,\zeta) \in UD\zeta_{n,\alpha}(q)$ then $f(z,\zeta) = L_a F(z,\zeta) \in S\zeta_n^*(q)$, where L_a is the integral operator defined by* (1).

Proof. From (1) we have

$$(1+a)F(z,\zeta) = af(z,\zeta) + zf'(z,\zeta), \quad z \in U, \zeta \in \overline{U}.$$

By means of the application of the linear operator D^n we obtain:

$$(1+a)D^{n+1}F(z,\zeta) = aD^{n+1}f(z,\zeta) + zD^{n+1}f'(z,\zeta)$$

or

$$(1+a)D^{n+1}F(z,\zeta) = aD^{n+1}f(z,\zeta) + D^{n+2}f(z,\zeta).$$

With the notation $\dfrac{D^{n+1}f(z,\zeta)}{D^n f(z,\zeta)} = p(z,\zeta)$, where

$$p(z,\zeta) = 1 + p_1(z,\zeta) + \cdots, \quad z \in U, \zeta \in \overline{U},$$

we have:

$$\frac{D^{n+1}F(z,\zeta)}{D^n F(z,\zeta)} = p(z,\zeta) + \frac{zp'(z,\zeta)}{p(z,\zeta)+a}.$$

If we denote $\dfrac{D^{n+1}F(z,\zeta)}{D^n F(z,\zeta)} = h(z,\zeta)$, with $h(0,\zeta) = 1$, we have from $F(z,\zeta) \in UD\zeta_{n,\alpha}(q)$ (see Definition 8) that:

$$h(z,\zeta) + \alpha\frac{zh'(z,\zeta)}{h(z,\zeta)} \prec\prec q(z,\zeta),$$

where $q(z,\zeta)$ is univalent in U with $q(0,\zeta) = 1$, $q'(0,\zeta) > 0$, and maps the unit disc U into the convex domain included in right half plane D.
From Theorem 1 we obtain
$h(z,\zeta) \prec\prec q(z,\zeta)$ or $p(z,\zeta) + \dfrac{zp'(z,\zeta)}{p(z,\zeta)+a} \prec\prec q(z,\zeta)$.
Using the hypothesis we obtain from Theorem 1 that
$p(z,\zeta) \prec\prec q(z,\zeta)$ or $f(z,\zeta) \in S^*\zeta_n(q)$.

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