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On supra α open sets and $S\alpha$-continuous functions[1]

R. Devi, S. Sampathkumar and M. Caldas

Abstract

In this paper we introduce and investigate a new class of sets and functions between topological spaces called supra α-open sets and $s\alpha$-continuous functions respectively.

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1 Introduction

In 1983, A.S. Mashhour et al. [2] introduced the supra topological spaces and studied s-continuous functions and s*-continuous functions. In 1987, M. E. Abd El-Monsef et al. [3] introduced the fuzzy supra topological spaces and studied fuzzy supra-continuous functions and obtained some properties and characterizations. In 1996, Keun [4] introduced fuzzy s-continuous, fuzzy s-open and fuzzy s-closed maps and established a number of characterizations. Now, we introduce the concept of supra α-open set, sα-continuous and investigate some of the basic properties for this class of functions.

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2 Preliminaries

All topological space considered in this paper lack any separation axioms unless explicitly stated. The topology of a space is denoted by τ and (X, τ) will be replaced by X if there is no chance for confusion. For a subset A of (X, τ), the closure and the interior of A in X with respect to τ are denoted by $cl(A)$ and $int(A)$ respectively. The complement of A is denoted by $X - A$.

Definition 1. [2] *A subfamily τ^* of X is said to be a supra topology on X if,*
(1) X, $\phi \in \tau^*$
(2) if $A_i \in \tau^*$ for all $i \in J$, then $\cup A_i \in \tau^*$
(X, τ^*) is called a supra topological space. The elements of τ^* are called supra open sets in (X, τ^*) and complement of a supra open set is called a supra closed set.

Definition 2. [2] *The supra closure o a set A is denoted by supra $cl(A)$ and defined as supra $cl(A) = \cap\{B : B$ is a supra closed and $A \subseteq B\}$.*
The supra interior of a set A is denoted by supra $int(A)$, and defined as supra $int(A) = \cup\{B : B$ is a τ supra open and $A \supseteq B\}$.

Definition 3. [2] *Let (X, τ) be a topological space and τ^* be a supra topology on X. We call τ^* a supra topology associated with τ if $\tau \subset \tau^*$.*

Definition 4. [2] *Let (X, τ) and (Y, σ) be two topological spaces. Let τ^* and σ^* are associated supra topologies with τ and σ respectively. Let $f : X \to Y$ be a map from X into Y, then f is a s-continuous function if the inverse image of each open set in Y is supra open in (X, τ^*).*

Definition 5. [1] *Let (X, τ^*) be a supra topological space. A set A is called supra semiopen set if $A \subseteq$supra cl(supra int(A)).*

3 Basic properties of supra α-open sets

In this section we introduce one new class of sets.

Definition 6. *Let (X, τ^*) be a supra topological space. A set A is called supra α-open set if $A \subseteq supra\ int(supra\ cl((A)))$.*

Theorem 3.1. *Every supra open set is supra α-open set.*

Proof. Let A be a supra open set in (X, τ^*). Since, $A \subseteq supra\,cl(A)$, then $supra\,int(A) \subseteq supra\,int(supra\,cl(supra\,int(A)))$. Hence

$$A \subseteq supra\,int(supra\,cl(supra\,int(A))).$$

The converse of the above theorem need not be true. This is shown by the following example.

Example 3.1. *Let (X, τ^*) be a supra topological space. Where $X = \{a, b, c\}$ and $\tau^* = \{X, \phi, \{a\}\}$. Here, $\{a, b\}$ is a supra α-open set, but not a supra open.*

Theorem 3.2. *Every supra α-open set is supra semiopen set.*

Proof. Let A be a supra α-open set in (X, τ^*). Therefore, $A \subseteq supra\,int$ $(supra\,cl(supra\,int(A)))$. It is obvious that, $supra\,int(supra\,cl(supra\,int(A)))$ $\subseteq supra\,cl(supra\,int(A))$. Hence $A \subseteq supra\,cl(supra\,int(A))$.

The reverse claim in the Theorem 3.2 in not usually true.

Example 3.2. *Let (X, τ^*) be a supra topological space. Where $X = \{a, b, c\}$ and $\tau^* = \{X, \phi, \{a\}\}$. Here, $\{b, c\}$ is a supra semiopen set, but not a supra α-open.*

Theorem 3.3. *(i) Finite union of supra α-open sets is always a supra α-open set.*
(ii) Finite intersection of supra α-open sets may fail to be a supra α-open set.

Proof. (i) Let A and B be two supra α-open sets. Then
$A \subseteq supra\, int(supra\, cl(supra\, int(A)))$ and $B \subseteq supra\, int(supra\, cl(supra\, int(B)))$. By [1] this implies, $A \cup B \subseteq supra\, int(supra\, cl(supra\, int(A \cup B)))$. Therefore $A \cup B$ is supra $\alpha-$open set.

(ii) Let (X, τ^*) be a supra topological space. Where $X = \{a, b, c, d\}$ and $\tau^* = \{X, \phi, \{a, b\}, \{b, c\}\}$. Here $\{a, b\}, \{b, c\}$ are supra α-open sets, but their intersection is a not supra α-open set.

Definition 7. *Complement of a supra α-open is a supra α-closed set.*

Theorem 3.4. *(i) Finite intersection of supra α-closed sets is always a supra α-closed set.*
(ii) Finite union of supra α-closed set may fail to be supra $\alpha-$closed set.

Proof. (i) This follows immediately from Theorem 3.5.
(ii) Let (X, τ^*) be supra topological space where $X = \{a, b, c, d\}$ and $\tau^* = \{X, \phi, \{a, b\}, \{b, c\}\}$. Here, $\{c, d\}, \{a, d\}$ are supra α-closed sets, but their union is not a supra α-closed set.

Definition 8. *The supra α-closure of a set A is denote by supra $\alpha cl(A)$ and defined as, supra $\alpha cl(A) = \cap\{B : B \text{ is a supra } \alpha\text{-closed set and } A \subseteq B\}$. The supra α- interior of a set is denoted by supra $\alpha int(A)$, and defined as, supra $\alpha int(A) = \cup\{B : B \text{ is a supra } \alpha\text{-open set and} A \supseteq B\}$*

Remark 3.1. *It is clear that supra $\alpha int(A)$ is a supra α-open set and supra $\alpha cl(A)$ is a supra α-closed set.*

Theorem 3.5. *(i) $X - supra\alpha int(A) = supra\, \alpha cl(X - A)$.*
(ii) $X - supra\, \alpha cl(A) = supra\, \alpha int(X - A)$.

Proof. (i) and (ii) are clear.

Theorem 3.6. *The following statements are true for every A and B.*
(1) $supra\, \alpha int(A) \cup supra\, \alpha int(B) = supra\, \alpha int(A \cup B)$
(2) $supra\, \alpha cl(A) \cap supra\, \alpha cl(B) = supra\, \alpha cl(A \cap B)$.

Proof. Obvious.

4 Sα-continuous functions

Definition 9. *Let (X, τ) and (Y, σ) be two topological spaces and τ^* be associated supra topology with τ. We define a function $f : (X, \tau) \to (Y, \sigma)$ to be a sα-continuous function if the inverse image of each open set in Y is a τ^* supra $\alpha-$open set of X.*

Theorem 4.1. *Every continuous function is sα-continuous function.*

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a continuous function. Therefore $f^{-1}(A)$ is a open set in X for each open set A in Y. But, τ^* is associated with τ. That is $\tau \subset \tau^*$. This implies $f^{-1}(A)$ is a supra open in X. Since supra open is supra α-open, this implies $f^{-1}(A)$ is supra α-open in X. Hence f is a $s\alpha$-continuous function.

Theorem 4.2. *Every s-continuous function is sα-continuous function.*

Proof. Obvious.

The converse of Theorems 4.1 and 4.2 may not be true. We can show this by following example.

Example 4.1. *Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a, b\}\}$ be a topology on X. The supra topology τ^* is defined as follows, $\tau^* = \{\phi, X, \{a\}, \{a, b\}\}$. Let $f : (X, \tau) \to (X, \tau)$ where, $f(a) = a, f(b) = c$ and $f(c) = b$. Since inverse image of $\{a, b\}$ is a supra α-open in X. Then f is a sα-continuous function. But the inverse image of $\{a, b\}$ is not a supra open set. So, f is not s-continuous and continuous function.*

Theorem 4.3. *Let (X, τ) and (Y, σ) be two topological spaces. Let f be a function from X into Y. Let τ^* be associated supra topology with τ. Then the followings are equivalent.*

(1) f is sα-continuous.

(2) The inverse image of closed set in Y is supra α-closed set in X.

(3) supra $\alpha cl(f^{-1}(A)) \subseteq f^{-1}(cl(A))$ for every set A in Y.

(4) $f(supra\,\alpha cl(A)) \subseteq cl(f(A))$ *for every set* A *in* X.
(5) $f^{-1}(int(B)) \subseteq supra\,\alpha int(f^{-1}(B))$ *for every set* B *in* Y.

Proof. (1) $\Rightarrow$ (2) : Let A be a closed set in Y, then $Y - A$ is open in Y. Thus, $f^{-1}(X - A) = X - f^{-1}(A)$ is supra α-open in X. It follows that $f^{-1}(A)$ is a supra α-closed set of X.
(2) $\Rightarrow$ (3): Let A be any subset of X . Since $cl(A)$ is closed in Y, then it follows that $f^{-1}(cl(A))$ is supra α-closed in X. Therefore, $f^{-1}(cl(A)) = supra\alpha cl(f^{-1}(cl(A))) \supseteq supra\alpha cl(f^{-1}(A))$.
(3) $\Rightarrow$ (4): Let A be any subset of X. By (3) we obtain, $f^{-1}(cl(f(A))) \supseteq supra\alpha cl(f^{-1}(f(A))) \supseteq supra\alpha cl(A)$ and hence $f(supra\alpha cl(A)) \subseteq cl(f(A))$.
(4) $\Rightarrow$ (5) : Let $f(supra\alpha cl(A)) \subseteq cl(f(A))$ for every set A in X. Then $supra\alpha cl(A) \subseteq f^{-1}(cl(f(A)))$, $X - supra\alpha cl(A) \supseteq X - f^{-1}(cl(f(A)))$ and $supra\alpha int(X - A) \supseteq f^{-1}(int(Y - f(A)))$. Then $supra\alpha int(f^{-1}(B)) \supseteq f^{-1}(int(B))$ Therefore $f^{-1}(int(B)) \subseteq supra\alpha int(f^{-1}(B))$, for every B in Y.
(5) $\Rightarrow$ (1): Let A be a open set in Y. Therefore,
$f^{-1}(int(A)) \subseteq supra\alpha int(f^{-1}(A))$, hence $f^{-1}(A) \subseteq supra\alpha int(f^{-1}(A))$.
But by other hand, we know that, $supra\alpha int(f^{-1}(A)) \subseteq f^{-1}(A)$. Then $f^{-1}(A) = supra\alpha int(f^{-1}(A))$. Therefore, $f^{-1}(A)$ is a supra α-open set.

Theorem 4.4. *If a map* $f : (X, \tau) \to (Y, \sigma)$ *is a* $s\alpha$*-continuous and* $g : (Y, \sigma) \to (Z, \gamma)$ *is continuous, then* $(g \circ f)$ *is* $s\alpha$*-continuous.*

Proof. Obvious.

Theorem 4.5. *Let* (X, τ) *and* (Y, σ) *be topological spaces. Let* τ^* *and* σ^* *be associated supra topologies with* τ *and* σ *respectively. Then* $f : (X, \tau) \to (Y, \sigma)$ *is a* $s\alpha$*-continuous map, if one of the following holds*
(1) $f^{-1}(supra\alpha int(A)) \subseteq int(f^{-1}(A))$ *for every set* A *in* Y.
(2) $cl(f^{-1}(A)) \subseteq f^{-1}(supra\alpha cl(A))$ *for every set* A *in* Y.
(3) $f(cl(B)) \subseteq supra\alpha cl(f(B))$ *for every set* B *in* X.

Proof. Let A be any open set of Y, if condition (1) is satisfied, then $f^{-1}(supra\alpha int(A)) \subseteq int(f^{-1}(A))$. We get, $f^{-1}(A) \subseteq int(f^{-1}(A))$. Therefore $f^{-1}(A)$ is a supra open set. Every supra open set is supra α-open set. Hence f is sα-continuous function.
If condition (2) is satisfied, then we can easily prove that f is sα-continuous function.
If condition (3) is satisfied, and A is any open set of Y. Then $f^{-1}(A)$ is a set in X and $f(cl(f^{-1}(A))) \subseteq supra\alpha cl(f(f^{-1}(A)))$. This implies $f(cl(f^{-1}(A))) \subseteq supra\alpha cl(A)$. This is nothing but condition (2). Hence f is a $s\alpha$-continuous function.

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R.Devi
Reader, Department of Mathematics,
Kongunadu Arts and Science College (Autonomous),
Coimbatore-29, Tamilnadu, India.
E-mail: rdevicbe@yahoo.com

S. Sampathkumar
Research Scholar, Department of Mathematics,
Kongunadu Arts and Science College (Autonomous),
Coimbatore-29, Tamilnadu, India
E-mail: sampath.twwi@gmail.com

M. Caldas
Departamento de Matematica Aplicada,
Universidade Federal Fluminense,
Rua Mario Santos Braga, s/n
24020-140, Niteroi, RJ, Brasil
E:mail: gmamccs@vm.uff.br